

Abstract

Title of the circuit: Op-Amp based Analog computer that solves second-order damped harmonic differential equations.

Theory:

Theoretical background: A second-order linear differential equation representing a mechanical spring-mass-damper system with a forcing function is defined as:

$$a \frac{d^2 x(t)}{dt^2} + b \frac{dx(t)}{dt} + cx(t) = f(t)$$

By feeding the acceleration term into successive integrator circuits made of op-amps, the circuit continuously evaluates the velocity and position of the system in real time.

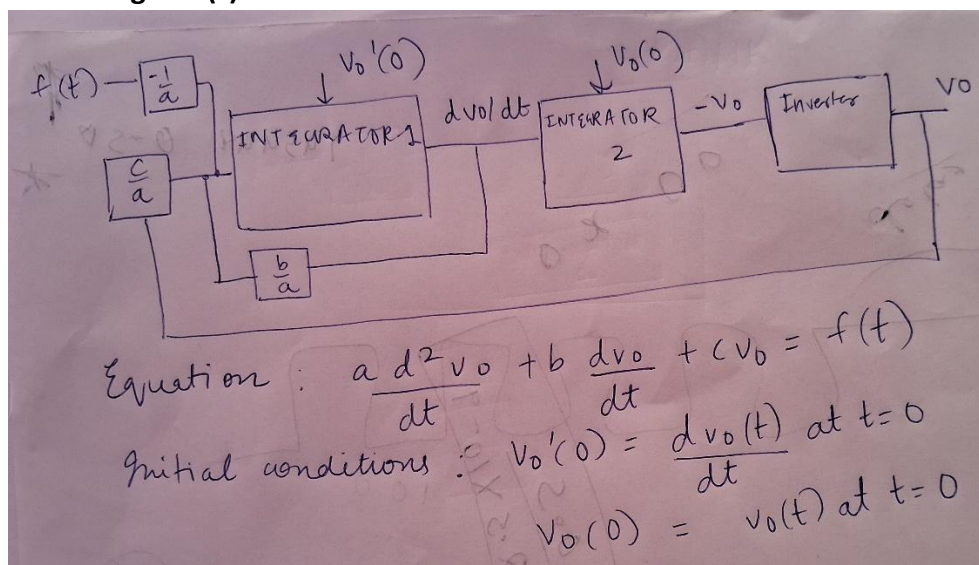
Key components: Op-Amps, Capacitor, Resistor

How the circuit functions:

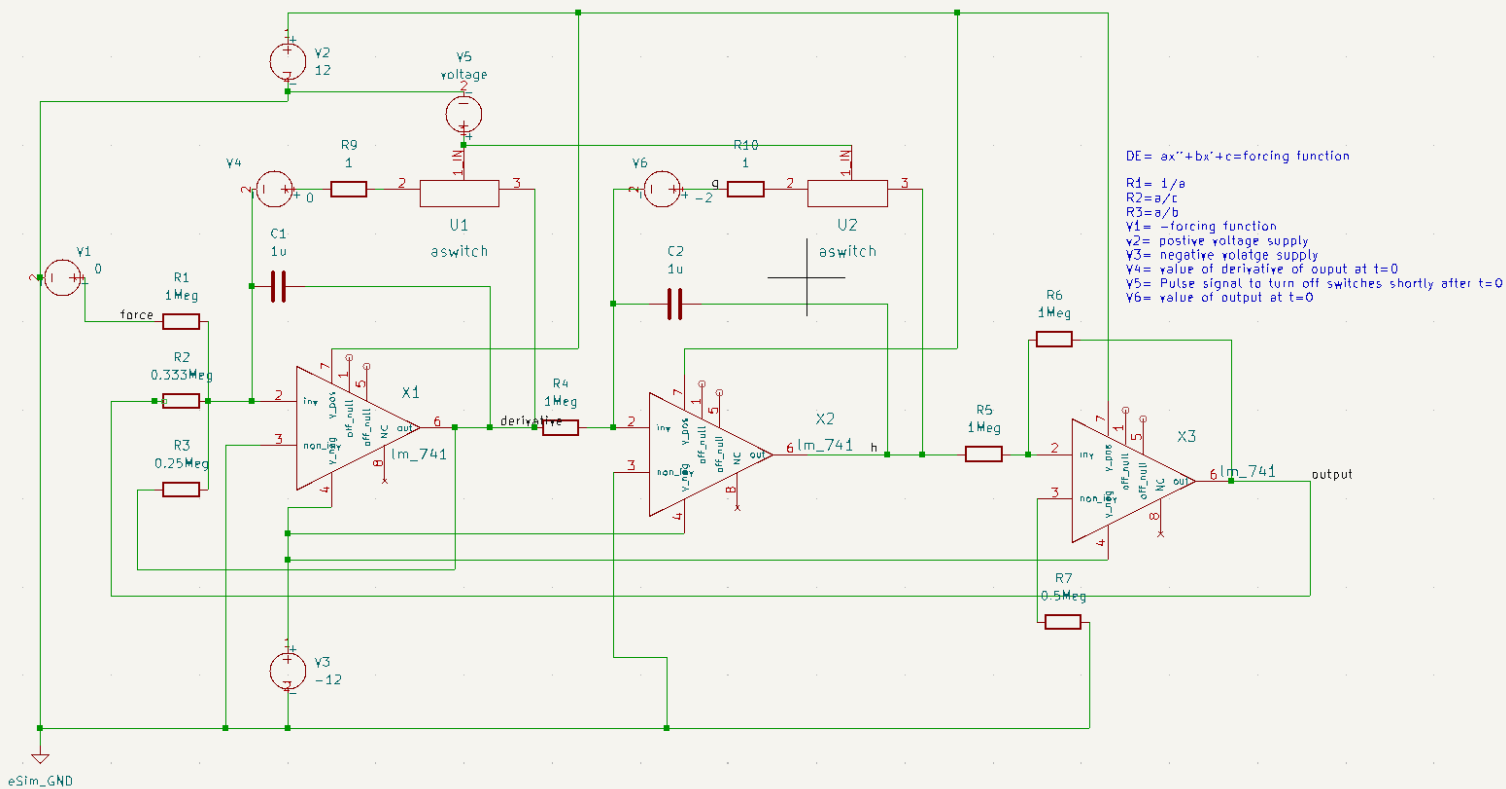
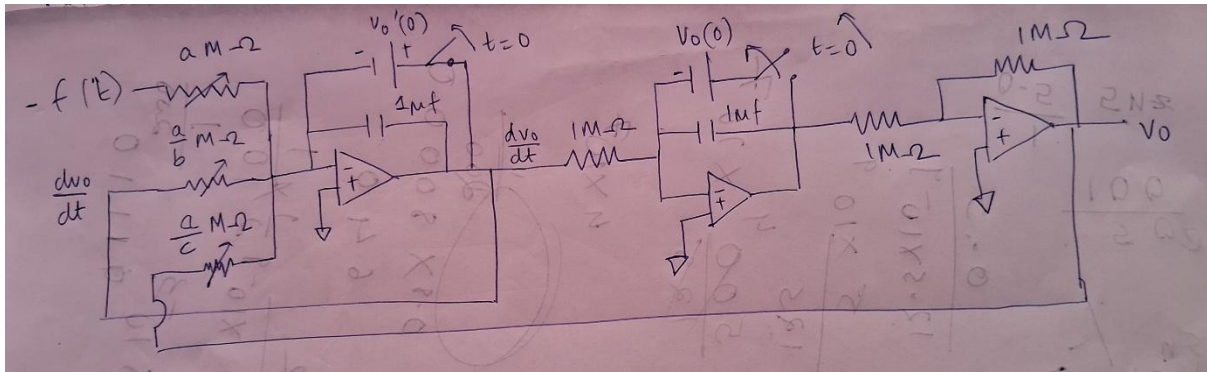
The circuit initializes when an external excitation voltage representing the forcing function is applied to the input. The Op-Amps instantaneously compute the algebraic sum of the system forces at the summing node. As current flows through the feedback capacitors of the integrator stages, the circuit outputs a continuous, time-varying analog voltage.

Varying the resistor values changes the differential equation's coefficients (a,b,c) and alters its feedback loop dynamics. The resulting transient response cleanly displays on an oscilloscope or simulation plot, capturing how the system transitions and approaches equilibrium.

Block Diagram (s)



Circuit Diagram(s) :



Results

The circuit has been simulated for 4 differential equations (DE). Each equation was solved using an online DE solver. The solution obtained was plotted in Desmos and compared with the Ngspice simulation.

$$1. \text{ DE} = \frac{d^2 v(t)}{dt^2} + 2 \frac{dv(t)}{dt} + v(t) = 10 \sin(4t), v(0) = -4, v'(0) = 1$$

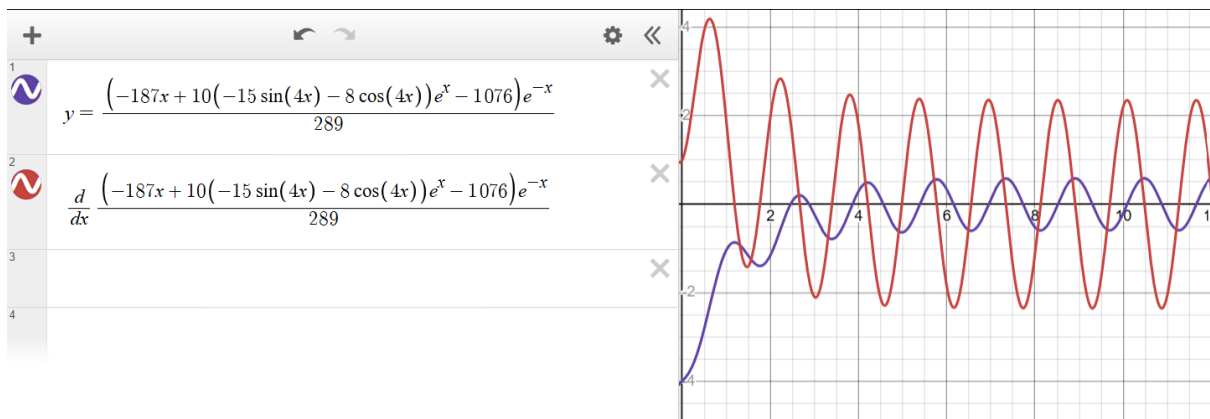
a. Solution:

Your input: solve $y(x) + 2y'(x) + y''(x) = 10 \sin(4x), y(0) = -4, y'(0) = 1$

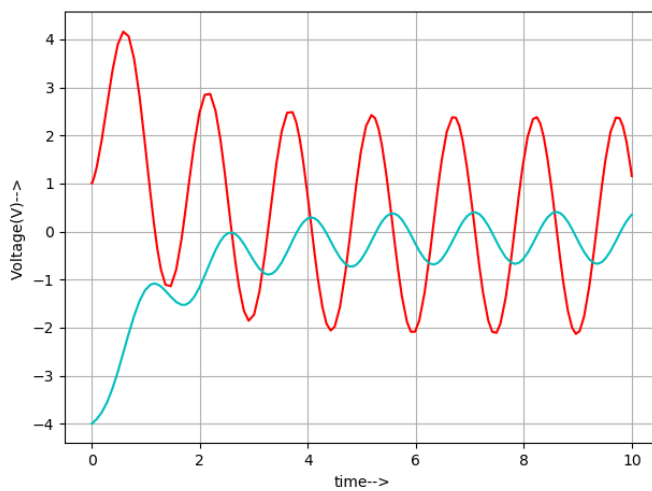
ANSWER

$$y(x) = \frac{(-187x + 10(-15 \sin(4x) - 8 \cos(4x))e^x - 1076)e^{-x}}{289}$$

b. Plot on Desmos



c. Ngspice simulation



2. DE = $\frac{d^2 v(t)}{dt^2} + 2 \frac{dv(t)}{dt} + 5v(t) = 0, v(0) = 2, v'(0) = 0$

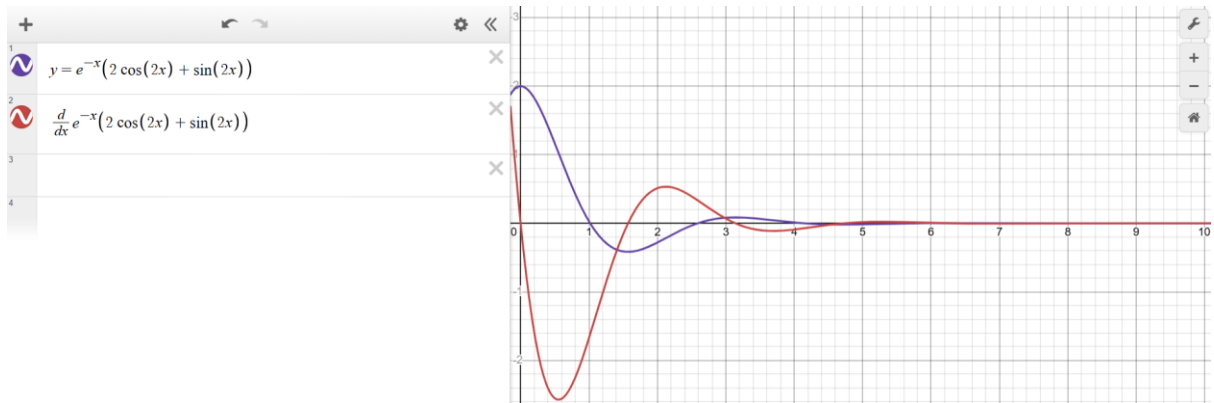
a. Solution

Your input: solve $5y(x) + 2y'(x) + y''(x) = 0, y(0) = 2, y'(0) = 0$

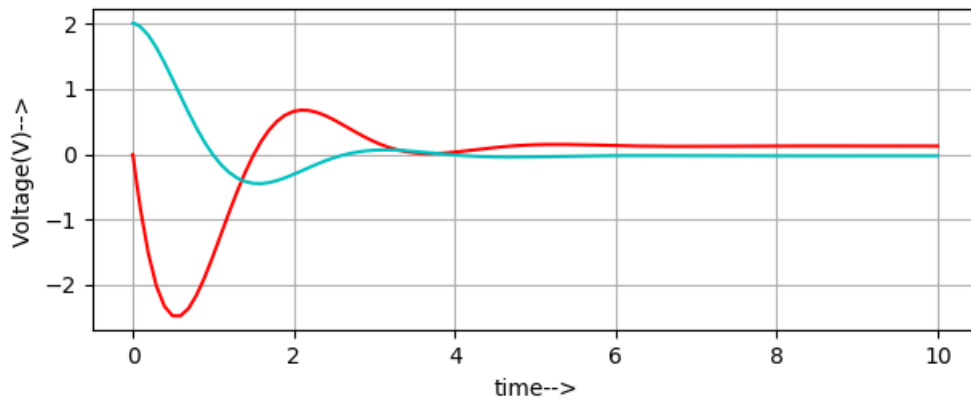
ANSWER

$$y(x) = (\sin(2x) + 2 \cos(2x)) e^{-x}$$

b. Plot on Desmos



c. Ngspice simulation



3. DE = $\frac{d^2 v(t)}{dt^2} + 6 \frac{dv(t)}{dt} + 9v(t) = 0, v(0) = 2, v'(0) = 0$

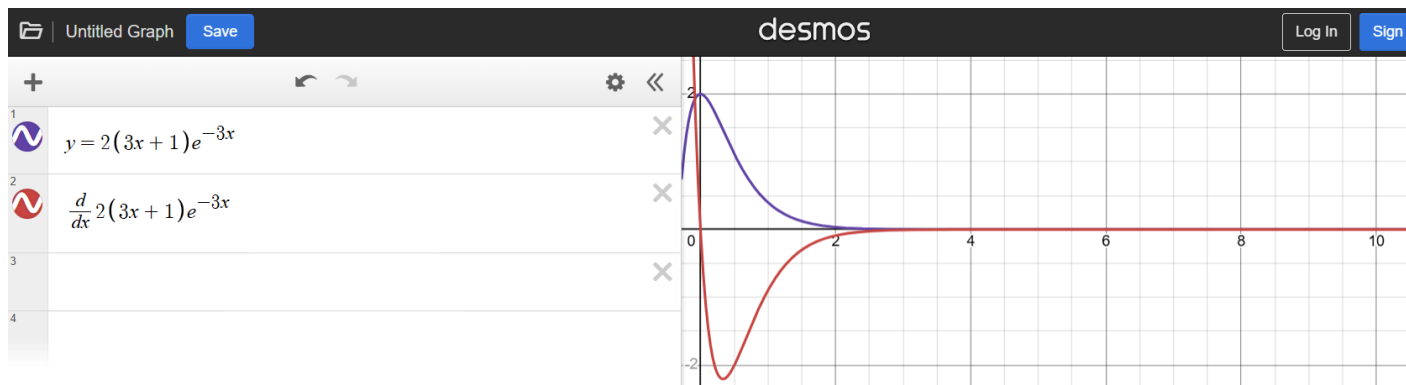
a. Solution

Your input: solve $9y(x) + 6y'(x) + y''(x) = 0, y(0) = 2, y'(0) = 0$

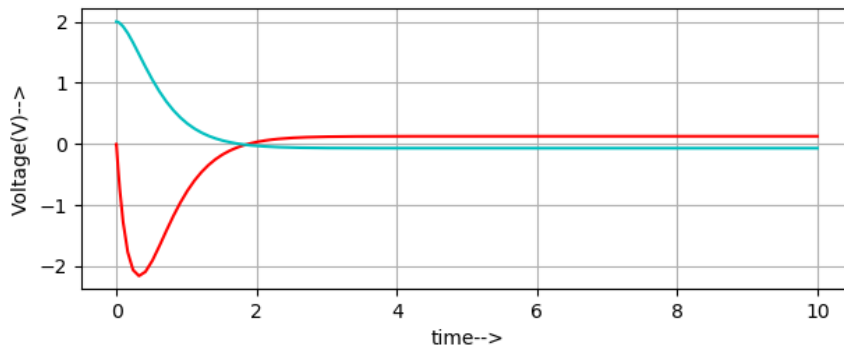
ANSWER

$$y(x) = 2(3x + 1)e^{-3x}$$

b. Desmos plot



c. Ngspice simulation



4. DE = $\frac{d^2 v(t)}{dt^2} + 4 \frac{dv(t)}{dt} + 3v(t) = 0, v(0) = 2, v'(0) = 0$

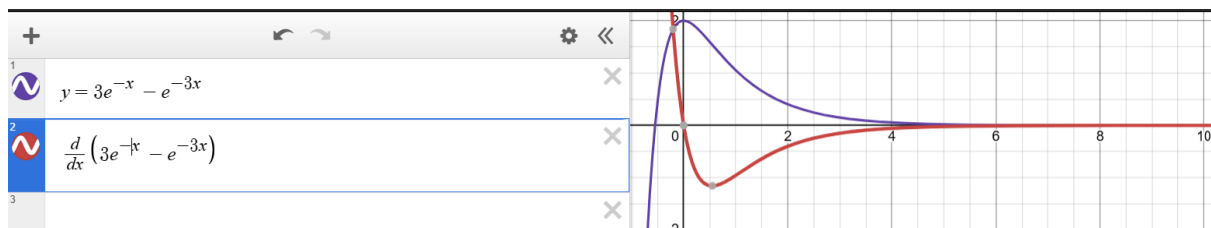
a. Solution

Your input: solve $3y(x) + 4y'(x) + y''(x) = 0, y(0) = 2, y'(0) = 0$

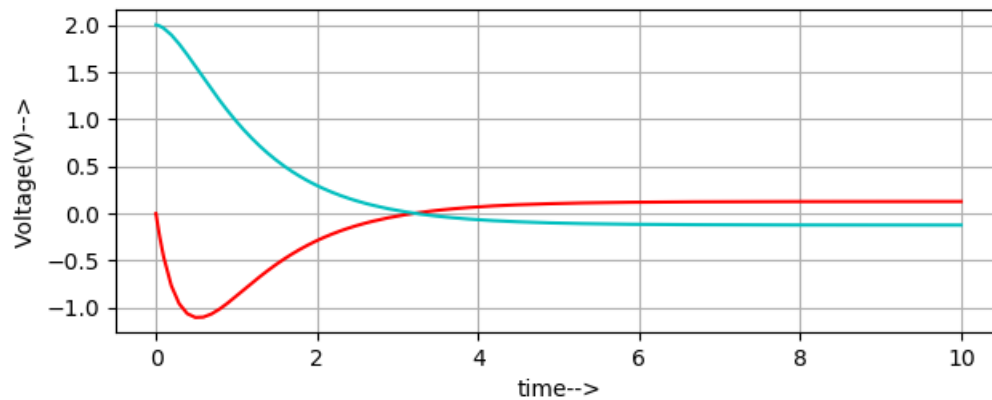
ANSWER

$$y(x) = 3e^{-x} - e^{-3x}$$

b. Desmos plot



c. Ngspice simulation



Research Paper/Journal/etc. :

Title : " A Study of the Applications of Analog Computers | IEEE Journals & Magazine "

Author : V.P. Kodali

Link : <https://share.google/QRrcpP2SYmCddJU7k>

Source/Reference(s) : Fundamentals of electric circuits by Sadiku and Alexander