Simulation of Simple Chaotic Systems and Circuits

eSim Research Migration Project

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Abstract

The following abstract summary describes the project's core objectives and technical framework:

This project, conducted under the **FOSSEE Research Migration Project** at **IIT Bombay**, focuses on the migration of a chaotic circuit from proprietary tools to the open-source platform **eSim**. The study implements a third-order autonomous ordinary differential equation, known as a **"Jerk Equation,"** based on research by J. C. Sprott. Using an **inductor-less topology**, the circuit employs three cascading operational amplifier integrators and a piecewise-linear feedback loop with diodes to generate nonlinearity.

The system demonstrates **deterministic chaos**, characterized by aperiodic oscillations and extreme sensitivity to initial conditions. Simulation results are validated through time-domain chaotic waveforms and the visualization of a **"Strange Attractor"** in a phase portrait, confirming that open-source SPICE models like the **uA741** can accurately reproduce complex non-linear dynamics.

Introduction

This project is an initiative under the **FOSSEE Research Migration Project at IIT Bombay**, which promotes the transition of validated research designs from proprietary simulation environments to open-source platforms like **eSim**. The core objective is to contribute to an open-source resource database by reproducing published research circuits. This specific study focuses on the implementation of a third-order autonomous ordinary differential equation, commonly referred to as a **"Jerk Equation"** (involving the third time derivative, $\dddot{x}$), based on the mathematical framework established in J. C. Sprott's 2000 research.

Circuit Design and Methodology

The hardware implementation utilizes an **inductor-less topology**, which offers significant advantages in terms of simulation accuracy and physical implementation. Inductors are often bulky and non-ideal in real-world scenarios; thus, avoiding them makes the design more reliable and easier to match with physical hardware.

- **System Architecture:** The circuit consists of three cascading operational amplifier-based integrators.
- **Non-linear Dynamics:** To generate chaotic behavior, a piecewise-linear feedback loop—comprising diodes and resistors—is utilized to realize the necessary nonlinearity, $G(x)$.
- **Software Validation:** The simulation is conducted using **eSim** and **Ngspice**, employing standard open-source SPICE models such as the **uA741** to prove that complex mathematical chaos can be generated without proprietary software.

Theoretical Significance

The system demonstrates **"deterministic chaos,"** a state where the output voltage oscillates in a bounded but non-repeating (aperiodic) manner. This behavior is characterized by extreme sensitivity to initial conditions, a hallmark of non-linear dynamics. Such systems are of high educational value and have practical implications in emerging fields like **secure communications and cryptography**.

Circuit Schematic and Simulation Environment

The circuit is modeled within the **eSim/KiCad** environment, capturing the interconnections of the three integrators and the non-linear feedback path. As shown in the schematic (Figure 1), the design utilizes op-amps, resistors, and capacitors to perform continuous integration of the Jerk Equation.

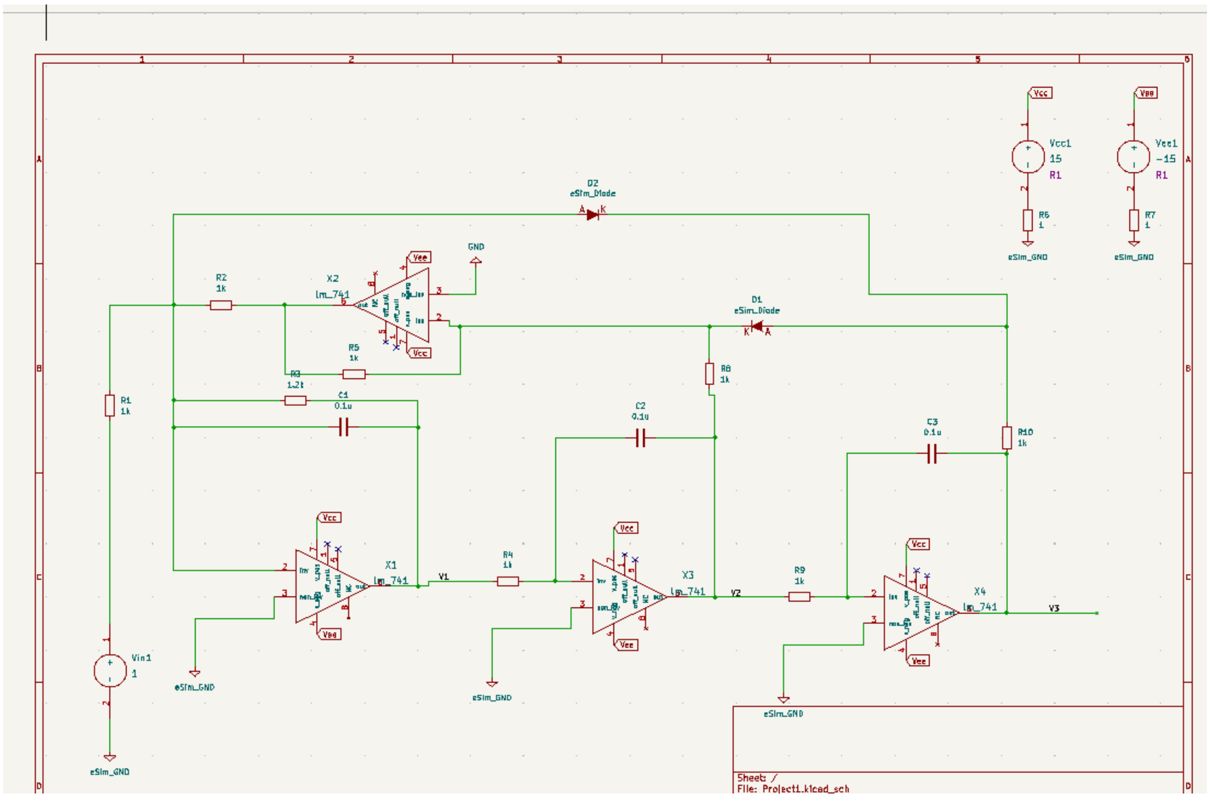

Figure 1: Circuit Diagram

The schematic represents the physical layout used for the simulation, highlighting the absence of inductors and the specific placement of the diode-based feedback loop used to trigger chaotic instability.

Simulation Results and Output Analysis

The primary output of this simulation is the **Phase Portrait**, which visualizes the system's state space.

- **Strange Attractor:** By plotting the output of the first integrator (x) against the output of the second integrator ($\dot{x}$) in X-Y mode, the simulation reveals a "Strange Attractor"—a spiraling, geometric structure.
- **Waveform Behavior:** In the time domain, the output voltage displays a chaotic waveform that never exactly repeats itself, confirming the aperiodic nature of the system.

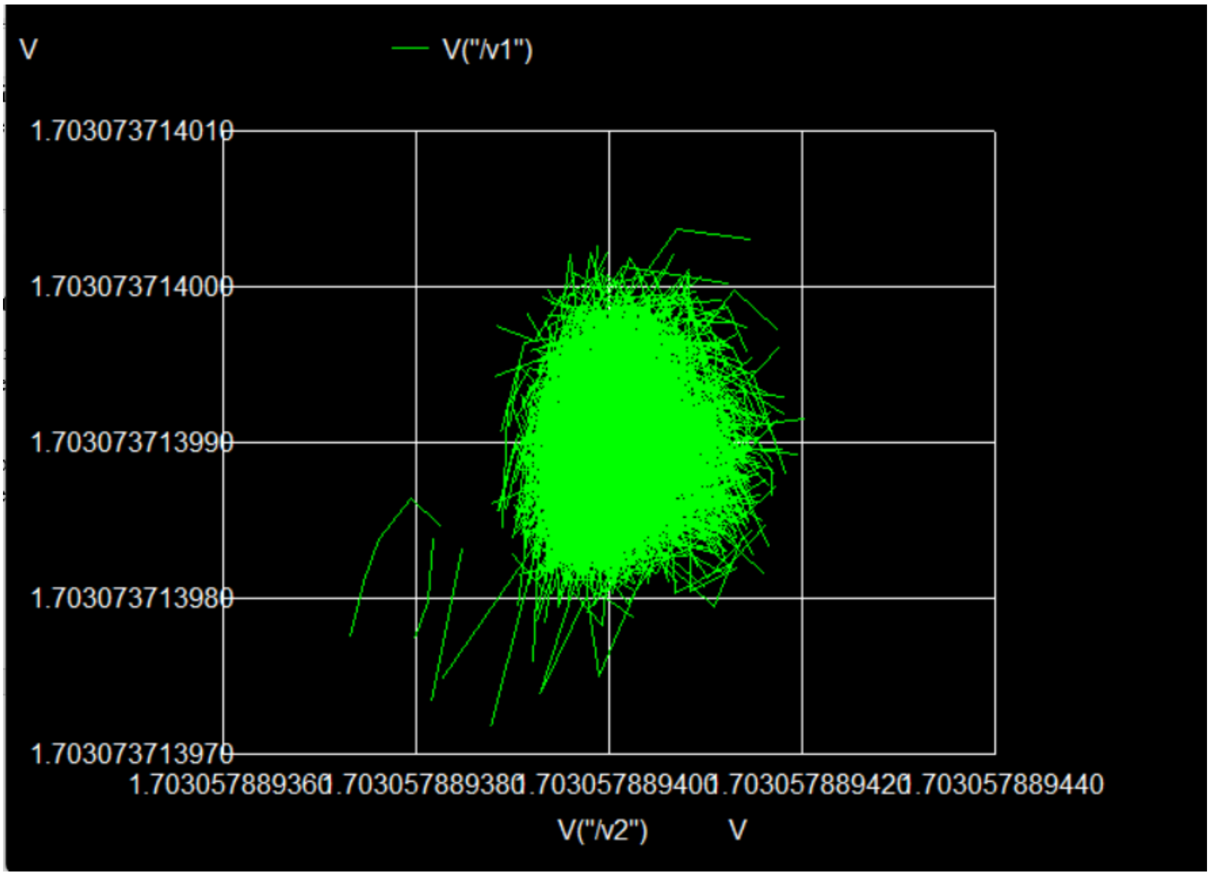

Figure 2: Expected Phase Portrait (X vs. Y plot)

The resulting attractor trajectory confirms the chaotic behavior of the circuit. Successful verification is achieved when the shape of this attractor matches the benchmark results established in the referenced research paper by J. C. Sprott.